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Translated by D.E.B.

PMM U.S.S.R., Vol.52, No.3, pp.328-333, 1988
Printed in Great Britain

0021-8928/88 $10.00+0.00

THREE-DIMENSIONAL DISTURBANCES IN A COMPRESSIBLE BOUNDARY LAYER*

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The propagation of three-dimensional disturbances from impulsive and harmonic sources in a compressible boundary layer on a plane plate is discussed. It is assumed that the Reynolds number tends to infinity. The field of the perturbed motion is obtained in the context of the linearized theory of the boundary layer with selfinduced paressure. The solution of the linearized equations is decomposed into Fourier integrals. When finding the inverse transformations, numerical and asymptotic methods are combined. A comparison is made with experimental data and calculations of the linearized Navier-Stokes equations. The theory of a boundary layer (BL) with selfinduced pressure /1, 2 / is useful for studying the BL instability in an incompressible fluid at high Reynolds numbers R, see e.g., /3-9/. At the same time, the asymptotic theory /1, 2/ predicts stability (in the limit as $R \to \infty$) of the supersonic BL with respect to plane disturbances propagating strictly along the flow, which is inconsistent with the well-known results for finite R, see e.g., /10, 11/. In the framework of asymptotic theory, however, the supersonic BL is unstable with respect to oblique waves (travelling at non-zero angles to the incoming flow) /12, 13/. It can therefore be expected that the packet of oblique waves of instability in the limit as $R \to \infty$ is qualitatively correctly described by the theory /1, 2/. (All the more, because at finite R, as the Mach number M_∞ of the incoming flow increases, the oblique waves become the most unstable, and their role is significantly increased in the supersonic BL /10, 11/). A packet of oblique waves is generated by any source which introduces serious three-dimensional perturbations into the boundary layer. In the present paper, such a source is taken to be injection and extraction via holes in the plate. The solution of specific problems assumes a detailed analysis of the influence of the Mach number M_∞ on the BL stability (in the limit as $R \to \infty$).

1. Time instability.

We start by analysing the dispersion relation (DR)

$$F(\Omega, k, m; M_\infty) = \Phi(\Omega) - Q(k, m; M_\infty) = 0 \tag{1.1}$$

$$\Phi = \frac{d\,\mathrm{Ai}(\Omega)}{d\zeta} I^{-1}(\Omega), \quad I = \int_\Omega^\infty \mathrm{Ai}(\zeta)\,d\zeta, \quad \Omega = \omega (ik)^{-2/3}$$

$$Q = (ik)^{1/3}(k^2 + m^2)/\sqrt{S}, \quad S = m^2 + (1 - M_\infty^2) k^2$$

obtained after linearizing the equations of the freely interacting compressible BL /14/ with respect to disturbances of the type $f(y)\exp(\omega t + ikx + imz)$ /3, 12, 13/ (x, y, z are dimensionless coordinates, measured respectively downstream, along the normal to the plate, and in the lateral direction, t is dimensionless time, M_∞ is the Mach number of the incident uniform flow, and $\mathrm{Ai}(\zeta)$ is the Airy function. In the case $S < 0$ we understand by the root

*Prikl.Matem.Mekhan., 52, 3, 423-430, 1988 in the expression for Q its branch on which $\sqrt{S} = i\sqrt{|S|}$. With this choice of the root sign, the DR (1.1) transforms with $m = 0$ and $M_\infty > 1$ into the DR for the direct plane waves in a supersonic BL.

We dwell first on the simpler analysis of time instability when k and m in (1.1) are assumed to be real, and the complex frequency $\omega = \sigma - i\omega_0$ has to be found from Eq.(1.1). For k and m which satisfy the inequality $S > 0$ (as is always the case when $M_\infty < 1$), we introduce the new variable $k' = \operatorname{sign} k\, |k|^{1/4}(k^2 + m^2)^{3/4}S^{-3/8}$. The DR (1.1) can then be rewritten as

$$\Phi(\Omega) = Q'(k'), \quad Q' = (ik')^{1/3}|k'| \tag{1.2}$$

A DR of type (1.2) has been considered, see e.g., /4, 7/, when studying plane disturbances in an incompressible fluid. Its properties are well-known. For instance, it has a denumerable number of roots $\omega_n'(k') = (ik')^{2/3}\Omega_n'(k')$, of which only the first is unstable: $\operatorname{Re}\omega_1'(k') > 0$ for $|k'| > k'_* = 1.0005$ (the maximum of $\operatorname{Re}\omega_1'(k')$ is reached at $|k'| = k_2'^* = 2.716$ and is equal to $\sigma_{0e} = 1.240$). Hence, by the chain of equations $\omega_n(k, m; M_\infty) = (ik)^{2/3}\Omega_n'(k') = (k/k')^{2/3}\omega_n'(k')$, the DR (1.1) likewise has only one unstable root with $n = 1$.

The relation

$$\omega_1(k, m; M_\infty) = (k/k')^{2/3}\omega_1'(k') \tag{1.3}$$

is a generalization of the well-known Squire transformation to the case of compressible flows in the limit as $R \to \infty$. (It was shown in /3/ that, with $M_\infty = 0$, relation (1.3) is the limiting form of this transformation as $R \to \infty$.) By means of it, we can easily trace analytically the influence of flow compressibility (expressed in the Mach number M_∞) on the boundary layer stability characteristics. For instance, the neutral curve in the (k, m) plane is given by the obvious equation $k' = k^{1/4}(k^2 + m^2)^{3/4}S^{-3/8} = k_*'$ (by symmetry, $\omega_1(-k, m; M_\infty) = \bar{\omega}_1(k, m; M_\infty)$ (the bar denotes the complex conjugate) and $\omega_1(k, -m; M_\infty) = \omega_1(k, m; M_\infty)$, so that it suffices to restrict ourselves to the range $k > 0$, $m > 0$). To find the maximum of $\operatorname{Re}\omega_1(k, m; M_\infty)$ with $M_\infty = \text{const}$, we introduce the variable $\beta = m/k$, after which (1.3) can be rewritten as

$$\omega_1(k, m; M_\infty) = f(\beta)\,\omega_1'(k[f(\beta)]^{-3/2})$$
$$f(\beta) = (1 + \beta^2)^{-1/2}[\beta^2 + (1 - M_\infty^2)]^{1/4}$$

and the problem reduces to finding $f_e = f(\beta_e) = \max f(\beta)$ for $0 < \beta < \infty$. Then, $\sigma_e = \max \operatorname{Re}\omega_1(k, m; M_\infty) = f_e \operatorname{Re}\omega_1'(k_2'^*)$, and the maximum σ_e is reached with $k = k_e = k_2'^* f_e^{3/2}$, $m = m_e = \beta k_e$. Analysis shows that $\beta_e = 0$ for $M_\infty < M_\infty^* = \sqrt{2}/2$ and $\beta_e = (2M_\infty^2 - 1)^{1/2}$, with $M_\infty > M_\infty^*$, whence

$$k_e = (1 - M_\infty^2)^{3/8}k_2'^*, \quad m_e = 0 \quad \text{for } M_\infty \leqslant M_\infty^*$$
$$k_e = (2M_\infty)^{-3/4}k_2'^*, \quad m_e = (2M_\infty^2 - 1)^{1/2}k_e \quad \text{for } M_\infty > M_\infty^*$$

Clearly, as M_∞ increases, an oblique wave with $m_e \neq 0$ becomes the most unstable when $M_\infty = M_\infty^* = 0.705$, which is in good agreement with the estimate $M_\infty^* = 0.6$–0.8 for finite Reynolds /10/. Here,

$$\sigma_e = \sigma_{0e}(1 - M_\infty^2)^{1/4}, \ M_\infty \leqslant M_\infty^*; \quad \sigma_e = \sigma_{0e}(2M_\infty)^{-1/2}, \ M_\infty > M_\infty^* \tag{1.4}$$

A more complete picture characterizing the time instability is given in Fig.1, where we show the isolines of the increment of the increase of $\operatorname{Re}\omega_1(k, m; M_\infty)/\sigma_{0e}$ for $M_\infty = 0.85$; the neutral curve is shown by the dot-dash curve. The domain with large values of the growth increment in the neighbourhood of the most unstable wave (k_e, m_e) is clearly pronounced (indicated by a cross). The plane disturbances with $m = 0$ are much weaker: $\max \operatorname{Re}\omega_1(k, 0; M_\infty) = \sigma_{0e}(1 - M_\infty^2)^{1/4} \to 0$ as $M_\infty \to 1 - 0$, whereas in accordance with (1.4), $\sigma_e \to \sigma_{0e}/\sqrt{2} \neq 0$ as $M_\infty \to 1$.

In the transonic domain $M_\infty \approx 1$, the theory of /1, 2/ is violated for two-dimensional flows. Whereas, the unstable three-dimensional oscillations in the (k_e, m_e) region, are described as before in the framework of the traditional scheme of /1, 2/ for all Mach numbers M_∞.

With $M_\infty > 1$, the case $S < 0$ has not been considered. It is of little interest and does not affect the results of the analysis concerned with finding the most unstable wave, since in this case (1.1) amounts to the DR for the direct plane waves in the supersonic BL /15/, which has no unstable roots.

2. An impulsive source. The above analysis of time instability is useful when solving the problem of the development of the disturbances generated by a source that acts impulsively on the boundary layer /7, 8/. Such a source generates a wave packet, the basic characteristics of which as $t \to \infty$ are determined by the properties of the DR on the most unstable wave /16, 17/.

We take as the source the injection-extraction via holes of diameter d^* in the plane plate. (For the equations of /14/ to be applicable, we require that the distance L^* between the forward edge and the centre of the hole be sufficiently large, so that the Reynolds number R, obtained from L^* and the incoming flow parameters, can be regarded as tending to infinity). The boundary conditions on the plate are

$$u = w = 0, \quad v = \delta v_0(t, x, z), \quad y = 0 \tag{2.1}$$

where u and w are the velocity vector projections on the x and z axes, and v is the vertical component of the velocity; $v_0 \equiv 0$ outside the hole $x^2 + z^2 \leqslant d^2/4$ (d is the dimensionless diameter) and $t \leqslant 0$ for $t \geqslant t_0$.

Assuming that the amplitude δ of the perturbations introduced is small, we linearize the equations of the freely interacting boundary layer /14/ with respect to this parameter, then, to solve the resulting linear problem, we apply a Laplace transformation with respect to time t and a Fourier transformation with respect to the x and z coordinates. For the function $A'(t, x, z)$, describing the instantaneous displacement of the streamlines in the thicker basic boundary layer /14/, we obtain (in the same way as in /3, 8/)

$$A' = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dm \int_{-\infty}^{\infty} dk e^{i(kx+mz)} \frac{1}{2\pi i} \int_{b-i\infty}^{b+i\infty} \frac{(ik)^{-1/3} v_0^*(\omega, k, m)}{F(\Omega, k, m; M_\infty)} e^{\omega t} d\omega \tag{2.2}$$

Here,

$$v_0^*(\omega, k, m) = \iint_{-\infty}^{\infty} dx\, dz e^{-i(kx+mz)} \int_0^{\infty} e^{-\omega t} v_0(t, x, z)\, dt$$

is the Fourier-Laplace transform of the function v_0 of boundary condition (2.1). Following /4, 7, 8/, we start the evaluation of integral (2.2) with the inverse Laplace transformation. We have

$$A' = \operatorname{Re}[A_c'(t, x, z)] \tag{2.3}$$

$$A_c' = \frac{1}{2\pi^2} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} dm \int_0^{\infty} dk \frac{v_0^*(\omega, k, m) \exp[\omega_n(k, m; M_\infty) t + ikx + imz]}{d\Phi[\Omega_n(k, m; M_\infty)]/d\Omega}$$

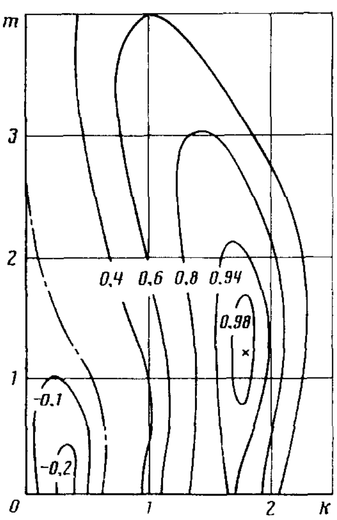

Fig.1

For the case of plane disturbances, we know how to estimate the contribution to the A_c' of (2.3) of the sum of all integrals, starting with the second (they describe the stable disturbances), see /4/, and the same method is easily extended to the three-dimensional case, and gives the same form $o(t^{-1})$ uniformly with respect to x and z.

To evaluate the integral with $n = 1$ remaining in (2.3), we devloped the special method of /7/, which is based on ideas of the saddle -point method. It was noted /8/ that, starting with the instant $t = 5$, good accuracy (a few percent) is obtained by evaluating the integral by the method of steepest descent. The main characteristics of the limiting form of the wave packet as $t \to \infty$ can be found in the context of Gauss's model /17/.

In short, the group velocity of the wave packet is given by the relations $U = -\partial \operatorname{Im} \omega_1/\partial k$, $V = -\partial \operatorname{Im} \omega_1/\partial m$ (where U and V are the projections on the x and z axes of the velocity of propagation of the perturbation amplitude maximum), which are evaluated at the point (k, m) with maximum increment of the growth σ_e. In the present case, with $M_\infty > M_\infty^*$, there are two such points: (k_e, m_e) and $(k_e, -m_e)$. By the DR symmetry with respect to m, the difference in the velocities U, V for these points can only consist in different signs of V (which would imply forking of the wave packet). Direct calculation gives $V = 0$ for all Mach numbers and $U = U_0(1 - M_\infty^2)^{-1/8}$ for $M_\infty \leqslant M_\infty^*$, $U = U_0(2M_\infty)^{1/4}$ for $M_\infty > M_\infty^*$, where $U_0 = 4.49$ is the group velocity of propagation of plane disturbances in an incompressible fluid /7/. Thus, though a pair of symmetric oblique waves becomes the most unstable when $M_\infty > M_\infty^*$, the disturbance still does not fork even in this case.

The Gauss model /16, 17/ can be extended to the case of the two most unstable waves. The final relations will not be quoted here because of their length. We note only that the isolines of the amplitude $|A_c'|$ in the neighbourhood of the maximum at $x = U_0 t$, $z = 0$, are ellipses as before; as M_∞ approaches M_∞^*, these ellipses are compressed along the z axis (this is connected with the vanishing of $\partial^2\omega_1(k_e, m_e; M_\infty^*)/\partial m^2$).

In Fig.2 we indicate by 1, 2, and 3 the dependences on the Mach number of σ_e/σ_{0e}, U/U_0, and λ/λ_0 respectively, where $\lambda = 2\pi/k_e$ is the characteristic wavelength in the packet, and $\lambda_0 = 2\pi/k_2'^*$ is its value in the incompressible fluid. These relations have a continuous derivative at the point of conjugation $M_\infty = M_\infty^*$.

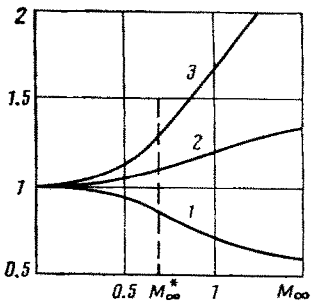

Fig.2

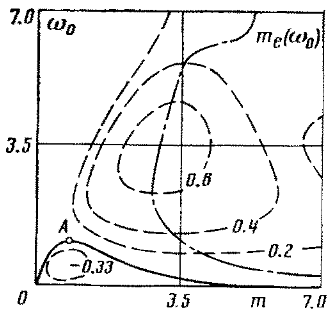

Fig.3

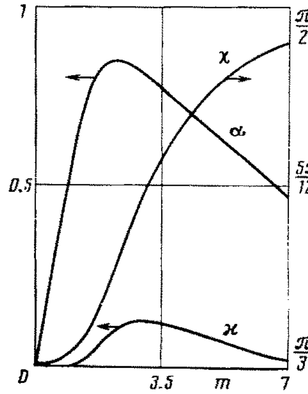

Fig.4

3. Three-dimensional instability. Let us analyse the three-dimensional instability qualitatively; for this, we put $\omega = i\omega_0$ in the DR (1.1) (ω_0 is the real frequency), and we assume as before that m is real. The complex quantity $k(m, \omega_0; M_\infty)$ then has to be found from the relation

$$\Phi(\Omega_0) = Q(k, m; M_\infty), \quad \Omega_0 = i^{1/3}\omega_0 k^{-2/3} \tag{3.1}$$

In the case of three-dimensional instability, the DR can no longer be reduced to a well-studied type for plane perturbations. It can be shown, however, that (3.1) also has a denumerable set of roots; only one, call it k_1, is unstable.

The isolines of the growth increment $\varkappa(m, \omega_0) = -\mathrm{Im}\, k_1(m, \omega_0; M_\infty)$ for $M_\infty = 2$ (it was at this Mach number that the experimental work of /18/ was performed, and the calculations in /11/, Sect.6.1, were made), are shown in Fig.3 (normalized to the maximum value $\varkappa = \varkappa_{ee} = 0.127$). Notice the pronounced maximum, reached on the oblique wave with $m = m_e = 3.02$ at $\omega_0 = 3.5$. (The frequency dependence of the side wave number m_e, corresponding to the most unstable wave with $\omega_0 = \mathrm{const}$, is shown by the dot-dash curve). The neutral curve with $\varkappa = 0$ is shown continuous in Fig.3.

There is an analytic representation for it which is too unwieldy to quote here, see /13/. We shall merely mention the coordinates of the point A, indicated by a small circle in Fig.3: $m = k_*'(2M_\infty)^{-3/4}(2M_\infty^2-1)^{1/2}$, $\omega_0 = \omega_*'(2M_\infty)^{-1/2}$ ($\omega_*' = 2.298$ is the neutral frequency for the direct plane waves in an incompressible fluid /3, 4, 7/). It is clear from this that, with $M_\infty > 1$, the size of the stable domain decreases as the Mach number increases.

As $m \to 0$ we have

$$k_1 = -\frac{m}{(M_\infty^2-1)^{1/2}} + \frac{M_\infty^4 m^5}{2(M_\infty^2-1)^{7/2}\omega_0^2}\left(1 + \frac{\sqrt{2}(1-i)m}{(M_\infty^2-1)^{1/4}\omega_0^{3/2}}\right) + O(m^7), \quad M_\infty > 1$$

Hence it follows that, for small m, the growth increment $\varkappa = O(m^6)$ is negligibly small even compared with the wave number $\alpha = |\mathrm{Re}\, k_1| = O(m)$. This is clearly illustrated in Fig.4, where we show the dependences on m of the increment $\varkappa$, of the wave number α, and of the angle of wave propagation $\chi = \mathrm{arctg}\,(m/\alpha)$ with $\omega_0 = 3.5$. It must be said that the type of behaviour of $\varkappa$ and α is in qualitative agreement with the calculations of /11/ from the Dan-Lien system and the experimental data /18/, with the exception that the direct plane wave in /11, 18/ has a non-zero growth increment and wave number. Moreover, $\chi \to \chi_0 = \mathrm{arctg}\,(\sqrt{M_\infty^2-1}) = \pi/3$ (χ_0 is the Mach angle) as $m \to 0$ (instead of $\chi \to 0$, as in /11, 18/).

In short, the ordinary three-deck scheme /1, 2/ predicts a pronounced maximum of the growth increment on the essentially three-dimensional (oblique) waves. This theory does not describe direct plane unstable waves in the supersonic BL. It seems that the growth increments of these waves amount to infinitesimal quantities with respect to the increments of the most unstable oblique waves as $R \to \infty$. Thus the main contribution to the wave packets is from the oblique waves described in the context of the ordinary scheme /1, 2/, while the contribution of the direct plane waves is negligible as $R \to \infty$.

4. Harmonic source. Starting from the instant $t = 0$, let the injection-extraction take place according to the harmonic law $v_0 = \sin(\omega_0 t)\, v_{00}(x, z)$ (thus, $v_0 = 0$ for $t < 0$). We have to introduce the initial instant $t = 0$ in order to define uniquely the weighting factors in the eigenfunctions, which are Tollmien-Schlichting/4, 9/ waves that increase exponentially downstream. The initial momentum when the source is triggered generates a wave packet, as previously described in Sect.2. As $t \to \infty$ a harmonic mode of oscillation is established. We have /9/

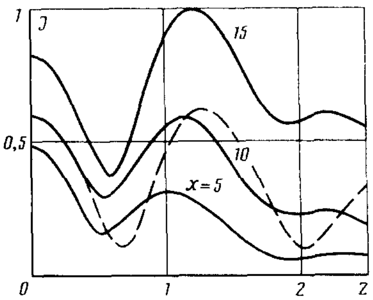

Fig.5

$$A' = \text{Im}\,[I_1(x, z)\, e^{i\omega_0 t}] + \text{Re}\,[I_2(x, z)\, e^{i\omega_0 t}]$$

$$I_1 = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dm e^{imz} \int_C \frac{(ik)^{-2/3} v_{00}^*(k, m)}{F[\Omega_0(k), k, m]} e^{ikx} dk$$

$$I_2 = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{(ik_1)^{-2/3} v_{00}^*(k_1, m)}{dF[\Omega_0(k_1), k_1, m]/dk} e^{\varphi(m, \omega_0; \text{tg}\,\alpha)x} dm$$

$$\Omega_0(k) = i^{1/3}\omega_0 k^{-2/3}, \quad \varphi = ik_1(m, \omega_0; M_\infty) + im\,\text{tg}\,\alpha, \quad \text{tg}\,\alpha = z/x$$

$$v_{00}^*(k, m) = \iint_{-\infty}^{\infty} v_{00}(x, z)\, e^{-i(kx+mz)}\, dx\, dz$$

To isolate the one-valued branch of the expression Q, we have to make cuts in the complex k plane (with fixed m) along the positive part of the imaginary k axis, and also along the rays $\text{Re}\,k = \pm m/\sqrt{M_\infty^2 - 1}$, $\text{Im}\,k > 0$ in the case $M_\infty > 1$, and along the rays $\text{Im}\,k \gtrless \pm m/\sqrt{1 - M_\infty^2}$, $\text{Re}\,k = 0$ when $M_\infty < 1$.

By the contour of integration C in the integral I_1 we mean any contour in the complex k plane (in general, dependent on m as a parameter), which passes above the first, but below all the other, roots of the DR (3.1) /4, 9/. Direct numerical calculation shows that the integral I_1 damps quite rapidly as $r = \sqrt{x^2 + z^2} \to \infty$ and with $r \approx 5$ its contribution to A' amounts to a few percent compared with the contribution of the integral I_2, which describes the superposition of the Tollmien-Schlichting oblique waves which increase downstream.

The behaviour of the integral I_2 as x increases depends on the exponential expression φ. An exponential rise is given by the quantity $\text{Re}\,\varphi = -\text{Im}\,k_1 = \varkappa$, which is discussed in detail in Sect.3. The fact along of the presence of the most unstable oblique wave (with $m = m_e \neq 0$) enables us to predict the qualitative picture of the development of the waves of instability at great distances downstream from the source: the disturbances increase most rapidly at an angle $\alpha = \alpha_e = \text{arctg}\,(|\,\partial\,\text{Re}\,k_1(m_e, \omega_0; M_\infty)/\partial m\,|)$ to the x axis /9/.

We fix $M_\infty = 2$ and $\omega_0 = 3.5$ (Fig.4). For these parameter values, $\alpha_e = 4.7^\circ$. In view of the small growth increments, however (recall that the maximum $\varkappa_e$ is at most only 0.127), the asymptotic estimates work badly even with $x = 5$–15 (the error amounts to tens of percent), as was also pointed out in /11/.

The results of numerical evaluation of the integral I_2 are shown in Fig.5 in the form of curves of $J = |I_2|/I_0$ against z ($I_0 = 0.217$ is a normalizing constant). The injection function was taken as $v_{00} = 4\pi^{-1}a^{-2}\exp(-4a^{-2}(x^2 + z^2))$ with characteristic source size $a = 2$ (the computations for $a = 1$ and $x = 10$ are indicated by broken curves). There is an obvious trend to preferential growth of the disturbances in a direction $\alpha_e = \text{arctg}\,(z/x)$, in qualitative agreement with the results of /11, 18/. Moreover, the actual shape of the curves with a sharp intermediate first maximum and a weak second maximum, is surprisingly similar to the results of /11, 18/. From Fig.5, the angle α_e can be estimated as 4-6°.

Let us emphasize that, according to our calculations, the perturbations in the three-dimensional packet of Tollmien-Schlichting waves increase downstream for fixed z in the range $0 \leqslant z \leqslant 2.5$, including strictly downstream (though the direct plane waves are stable in the supersonic BL).

The author thanks O.S. Ryzhov for useful discussions and A.M. Tumin for his interest.

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Translated by D.E.B.

PMM U.S.S.R.,Vol.52,No.3,pp.333-343,1988
Printed in Great Britain

0021-8928/88 $10.00+0.00

ON NON-STATIONARY MOTIONS OF LOCAL INHOMOGENEITIES IN A PSEUDOFLUIDIZED LAYER*

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The growth (collapse) of a moving local inhomogeneity in the concentration of particles in a pseudofluidized layer is investigated. The inhomogenity is modelled using a spherical packet of particles /1-3/. The mass of the packet and the distribution of the particles throughout its volume remain constant. The density of the solid phase is assumed to be large compared with the density of the fluidizing fluid while the interaction between the phases is assumed to be linear with respect to the velocity of the relative motion of the phases. The simplest model, where there is no exchange between the particles in the packet and the particles in the layer, is analysed.

As a result of the approximate solution of the problem on the motion of a packet of variable radius, a system of equations is obtained which relates the change in the size of the packet with the velocity of its motion in the layer and the rate of circulation of the disperse phase in it. The velocity and pressure fields inside and outside the packet are found and the stationary states of the system are determined. It is shown that, unlike the case of bubbles where there is always a unique stationary state /4, 5/, the number of stationary states of the packet can vary depending on the physical parameters of the pseudofluidized system.

**Prikl.Matem.Mekhan.*,52,3,431-443,1988